On the Sources and Implications of Carnap's *Der Raum*

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Abstract

Der Raum, Carnap's earliest published work, finds him largely a follower of Husserl. In particular, he holds a distinctively Husserlian conception of the synthetic a priori—a view, I will suggest, paradigmatic of what he would later reject as "metaphysics." His main purpose is to reconcile that Husserlian view with the theory of general relativity. On the other hand, he has already broken with Husserl, and in ways which foreshadow later developments in his thought. Especially important in this respect is his use of Hans Driesch's *Ordnungslehre*.

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Carnap's first publication, *Der Raum* (1922), remains relevant to controversies which continue today, and is perhaps still to be taken seriously. My own interest, however, is in what it reveals about Carnap's background and subsequent development. The bulk of this paper will of necessity be devoted to the former, since the latter would require extensive interpretation of later Carnap. Ultimately, however, the forward view is more important: if Carnap had never written anything beyond *Der Raum*, we would have little interest in his background. So let me begin with a few, necessarily sketchy words about forward-looking implications.

First, Carnap's later thought is based on the rejection of traditional "metaphysics." To understand him, we must understand what it is he rejects. *Der Raum*, however, comes before the anti-metaphysical turn. One way to see this is

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to note that, for later Carnap, rejection of metaphysics goes along with rejection of the synthetic a priori. The very meaning of "logical empiricism" is that all judgments are either analytic or empirical; synthetic a priori judgments "do not occur at all according to the view of constitution theory" (Carnap, 1974 [1928], §106L, p. 148). As Carnap himself reports, however: "in [*Der Raum*] I took an empiricist position in regard to *physical* geometry, while following Kant and Husserl in considering our knowledge of the *topological* features of the '*Anschauungsraum*' ... as synthetic a priori" (Carnap, 1963a, p. 957). I will show here that this assessment of *Der Raum* is accurate. Carnap at this point is a follower of Husserl—and of Kant, as well, *as interpreted by Husserl*.[1] My suggestion, then, will be that the "metaphysics" rejected by the later Carnap is paradigmatically Husserlian phenomenology.

Second, however, *Der Raum* already exhibits deviations from Husserl, foreshadowing the bigger changes to come. To lend technical precision to Husserl's points and to explain his own departures, Carnap draws on other philosophers and mathematicians, such as Dingler, Hilbert, Killing, Klein, Pasch, Poincaré, Russell, Weyl, and, above all, Driesch. Driesch's influence is most apparent, I will argue, in the appearance of freely chosen demands or postulates (*Forderungen*) within the epistemology of *Der Raum*. As I will briefly suggest towards the end of the paper, this marks an early version of the "voluntarism" which was both to turn Carnap against Husserl and to drive much of his later development.

1 Husserlian question, Husserlian answer

Der Raum aims to settle a question concerning "sources of knowledge" (*Erkenntnisquellen*) about space (Carnap, 1922, Introduction, p. 5). Given Kant's dominance over German-speaking philosophy at the time, this Kantian terminology[2] might seem to offer little clue as to the background of Carnap's inquiry.

[1]On Husserl as a Kantian, see further below. The importance of Husserl in *Der Raum* has been recognized many times. Michael Friedman, although associated with the reading of Carnap as a reaction to neo-Kantianism (a reading with which, as should already be apparent, I disagree), has nevertheless pointed out Husserl's role in various places: see (1994), p. 48; (1995), pp. 51–8; (2000), pp. 67–8, 93 n. 128. See also Sarkar (2003); Mormann (2007), pp. 46–7; Ryckman (2007), p. 103; and, at great length and with great vigor, Haddock (2008). Alan Richardson, in contrast, calls Carnap in *Der Raum* "an unabashed, if unorthodox, neo-Kantian" (1998, p. 139), while Yemima Ben-Menahem argues that *Der Raum* "echoes Poincaré in the problems it poses, the solution it reaches, and the character of its arguments" (2006, p. 181). I will make some remarks about these competing interpretations below.

[2]See e.g. Kant (1990), A260/B316. This work is no. 125 in the bibliography (Literatur-

But, in fact, the question is revealing.

For Kant, questions about *Erkenntnisquellen* arise because he holds that our knowledge has two such sources, intuition (*Anschauung*) and thought.[3] But many of his successors, including both the German Idealists and the neo-Kantian schools of the late 19th and early 20th century, abandoned this view. As the Marburg neo-Kantian Paul Natorp explains:

> The subsequent philosophy which emanates from Kant, including the present, no less than "orthodox," neo-Kantian movement, has more and more taken offense at the dualism of pure intuition and pure thinking and finally broken with it decisively.[4]

The B edition of the First Critique was thought to show traces of a move in this direction, since Kant says there that all synthesis has its origin in the understanding, and hence finds the understanding at work in the givenness of the manifold itself.[5]

But there were two groups among whom the question of *Erkenntnisquellen* was alive and well. First, philosophically minded mathematicians and physicists, including Frege (whose life project was to answer this question with respect to arithmetic) and many others cited by Carnap—for example, Pasch, Killing, and Hausdorff. These generally dealt with an unsophisticated, or anyway unreflective, version of the question. Typical is Frege's treatment in the *Grundlagen* (1884), in which he barely pauses to say what he means by such terms as "analytic," "synthetic," "intuition," and "logic." Detailed epistemological discussions are found, instead, among the second group: intellectual descendants of Brentano.[6] Questions about "the origin of our concepts" exercise both

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Verzeichnis) of *Der Raum*. I give that information in the References section below in the form: RLV 125.

[3]See Kant (1990), A294/B350, and see also A271/B327. (Kant sometimes gives other lists: see A38/B55, A97, A299/B356.)

[4]Natorp (1910), p. 2. (Natorp refers here only to *pure* intuition; but, as for empirical intuition, he regards it as the infinite goal of *Erkenntnis*, rather than as a source of it: see pp. 273–4, 277.) Friedman, well aware of this feature of neo-Kantianism, considers it "puzzling" that Carnap raises a traditional question about *Erkenntnisquellen* in *Der Raum* (2000, p. 66). He solves the puzzle, in fact, by invoking Husserl—but without, I think, appreciating the depth of Husserl's influence, and based on an interpretation of Husserl which I would not accept.

[5]See Kant (1990), B129–30; Natorp (1910), pp. 275–6.

[6]See, on this school, Smith (1994). Smith agrees in assigning Carnap to the "Austrian" tradition, while attaching the Marburg neo-Kantians to the "German" tradition which runs through Hegel (see especially p. 9 n. 3).

Brentano and his many students.[7] But Brentano himself, who had little regard for Kant, traces such questions to Locke and Leibniz. Husserl, on the other hand, came to consider himself a Kantian.[8] Husserl's interpretation of Kant is questionable and selective, but the same can be said of Natorp or Rickert. And one of Husserl's ways of assimilating his views to Kant's is precisely to identify these questions about the origin of concepts with Kantian questions about *Erkenntnisquellen*. In short, it is wrong to ask (as, e.g., Sarkar and Haddock do) whether Carnap is Kantian *or* Husserlian. He is a Husserlian Kantian, and would be marked as such by the form of his question, even if he never mentioned Husserl, and even if the two had never met in person.

But Carnap does indeed mention Husserl, and the two did eventually meet. Only the date of their first meeting is uncertain. We know, from a report of Ludwig Landgrebe, that Carnap participated in Husserl's advanced seminars from Summer 1924 through Summer 1925.[9] Moreover, when Landgrebe later considered doing his habilitation in Prague, he told Husserl that, though he would have his uncle write to Oskar Kraus there, he was worried because Kraus had attacked Husserl so heavily in print. "But," he continues,

> Carnap—who, despite his totally different approach [*Einstellung*], still has high regard for phenomenology, and who will likely still remember me from his time in Freiburg—is now also there. I had many discussions with him in that period. I believe I can likely risk writing to him myself at the same time.[10]

This does nothing, of course, to show that Carnap had personal contact with Husserl by 1921. Haddock has searched in vain for documentary evidence of such (2008, p. ix). But Haddock also considers it nearly certain—and I agree—that Carnap at least attended Husserl's lectures. Although Carnap wrote his dissertation under Bruno Bauch at Jena, he had already moved to Buchenbach, near Freiburg, in 1919.[11] Landgrebe, on the other hand, arrived in Freiburg and

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[7] As Smith notes: (1994), p. 107.

[8] See, e.g., Husserl (1976 [1913]), §62, p. 118/Hua 3.1:133,27–30. (I cite this work in the form x/y, where x is the section and page numbers in the first edition and y is the page and line numbers in the Husserliana edition.)

[9] See Schuhmann (1977), p. 281. Carnap could not have participated *after* 1925, the year he left for Vienna.

[10] Landgrebe to Husserl, November 11, 1932, in Schuhmann & Schuhmann (1994), pt. 4, p. 298,19–23. (I owe my awareness of this letter to Haddock: see [2008], p. 2 n. 8.)

[11] See Gabriel (2004), p. 18 n. 29.

met Husserl for the first time, as a twenty-one year old student, in the summer of 1923.[12] So if Carnap had pre-1924 dealings with Husserl, Landgrebe was in no position to describe them with certainty to Schuhmann. Summer 1924 should be regarded as merely as an upper bound for Carnap and Husserl's first meeting.

The evidence for *textual* influence by 1921 is, in any case, unambiguous. Carnap answers his distinctively Husserlian question in explicitly Husserlian terms. Parceling out knowledge between empirical, synthetic a priori, and analytic, he, like Husserl, expresses reservations about the terminology.[13] Husserl does use these terms, "in order to let historical parallels resonate," but "only as equivalents for other terms given together with them" (1976 [1913], Introduction, p. 6/Hua 3.1:8,23–6). Carnap follows the same policy, in part by leaning on Husserl. Our knowledge is "analytic," he explains, insofar as it derives from "formal ontology in Husserl's sense," and he distinguishes between "a priori" and "a posteriori" in terms of Husserl's characteristic distinction between essence, known through eidetic insight (*Wesenserchauung*), and "matters of fact."[14]

Husserl is not the only one Carnap invokes for this purpose: he also draws on Hans Driesch. Today mostly remembered, if at all, as a neo-vitalist, Driesch was then known for his general work on logic, metaphysics, and epistemology. Carnap gives him prominent billing both in *Der Raum* and in the *Aufbau*, with special attention to his logico-epistemological book, the *Ordnungslehre* (1912). In *Der Raum*, Carnap consistently supplements Husserlian glosses with Drieschian ones (1922, §I, p. 8; §II, p. 22; §IV, p. 60; §V, p. 63). And Driesch's influence is significant, as we will see. Still, in main outline, Carnap's approach is Husserlian. To appreciate this, we need only compare Husserl's own answer to the same question about space.

Husserl returned to that question many times: new thoughts on it mark the developments of his final period. His basic approach, however, which occurs already in the *Logische Untersuchungen*, is that it depends what you mean by "space."[15] Space qua "world-space," "the well-known order-form of the world

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[12] See Landgrebe (1988), 20–21.

[13] See, respectively, Carnap (1922), §V, p. 63; Husserl (1976 [1913]), Introduction, pp. 5–6/ Hua 3.1:8,15–22.

[14] Carnap (1922), Introduction, p. 6; §II, pp. 22–3; §IV, p. 61; §V, pp. 64–5. Cf., e.g., Husserl (1976 [1913]), Introduction, pp. 3–4/Hua 3.1:6,3–18.

[15] See Husserl (1975 [1900/1913]), Prolegomena §70, pp. 250–53. This passage is specifically cited by Carnap (1922, Literatur-Hinweise, p. 78 [note to p. 7]).

of appearance," is Euclidean: in this sense, "the talk of 'spaces' for which, e.g., the parallel axiom doesn't hold, is . . . an absurdity."[16] But the "categorial form" of this world-space can also be regarded on its own, as a form under which spatially related objects fall, leaving the objects themselves "fully indeterminate with respect to matter" (Husserl, 1975 [1900/1913], p. 250,23–4). Space in *this* sense, as a formal structure, is just one among many possible "spaces," and this allows for the theory of manifolds "which has grown out of generalizations of geometrical theory," including in particular "the theory of n-dimensional manifolds, whether Euclidean or non-Euclidean" (p. 252,1–4).

But if higher dimensional and/or non-Euclidean spaces are formally possible, why shouldn't the world-space instantiate one? This touches on the distinction, in Husserl's later terminology, between formal and material eidetic necessity. A material essence or "eidos" prescribes what is necessary to some genus or species of objects. Since every object belongs to a hierarchy of species and genera, each falls under a hierarchy of material eidetic laws, culminating in the laws of a "regional ontology," which apply to it by virtue of its highest genus.[17] But higher laws encompass lower ones, insofar as they predelineate the possibilities for differentiation. So material eidetics and regional ontology are basically the same.

Formal eidetics, in contrast, is about the form of a region in general. Every region contains parts and wholes, objects and properties, classes and members, and so forth: the existence and nature of such structures are matters of formal eidetic truth. Husserl identifies the discipline of formal ontology, which is concerned with such truths, with formal logic (broadly construed to include mathematics, set theory, mereology, and so on). So formal-logical or "analytic" truths concern structures in abstraction from what they are struc-

[16] Husserl (1975 [1900/1913]), p. 252,20–23. Note that this and many other clear statements rule out Haddock's contention (2008, 31) that Husserl had abandoned the idea that physical space is necessarily Euclidean as early as 1892. Haddock cites Husserl's letter to Brentano of December 12, 1892 (Schuhmann & Schuhmann, 1994, pt. 1, pp. 10,24–11,14). All Husserl actually says there is that he no longer considers denial of the parallel axiom to be inconsistent, and therefore admits that, "despite superficiality and errors in many particulars," Riemannian-Helmholtzian theories of space "do not lack a valuable core." This is completely compatible with Husserl's later position as I describe it.

[17] A *region* is a complex of highest genera which go together (for example, the highest genera of an object and of one of its properties will always belong to the same region). See Husserl (1976 [1913]), §16, pp. 30–31/Hua 3.1:36,21–30 for a more precise definition. Examples of regions include (physical) nature, the psychological region, and the phenomenological region of pure consciousness.

tures *of*, ways in which things of any kind whatsoever might be related—or, as Husserl also says, "modifications [*Abwandlungen*] of the empty something" (1976 [1913], §14, p. 28/Hua 3.1:33,17). This conception of the formality of logic and set theory (which was always to underwrite Carnap's strong logicist convictions about mathematics, and which also ultimately gave rise to our own semantic conception of logical validity as "truth under every interpretation") is distinctively Husserlian.[18] And the categorial form *Euclidean three-manifold*—defined in the *Logsiche Untersuchungen*, recall, as "fully indeterminate [*völlig unbestimmt*] with respect to matter"—is obviously formal in this Husserlian sense. Our knowledge about it is thus obviously formal eidetic, i.e., analytic.

If a non-Euclidean world-space is formally unobjectionable but nevertheless absurd, then it must violate regional, *material*-eidetic constraints. Such is the force of Husserl's remark, in (1976 [1913]), that the formal essence, *Euclidean* [*three-*]*manifold* is a "formal universalization" of the material essence *space* (§13, p. 27/Hua 3.1:32,15–18). The point is that the matter-of-fact world-space (a high order, categorially structured object belonging to the region of nature) is necessarily, by virtue of its material essence, an instance of the formal category in question. The knowledge of this necessary truth about the natural world is "synthetic a priori": based on material-eidetic insight. So while space, on one understanding, is a formal structure, the knowledge of which is analytic, it is also, on a different understanding, an empirical, physical structure, about which, finally, we also know synthetic a priori (material-essential) truths.

Carnap in *Der Raum*, similarly, explains that space, in one sense, is a formal structure, which might order objects of any kind: an "order-system [*Ordnungsgefüge*] ... of relations, not between determinate objects of a sensible or non-sensible realm, but rather between wholly indeterminate [*ganz unbestimmten*] relation-members" (Carnap, 1922, Introduction, p. 5). He assigns our knowledge about it, in that sense, to formal ontology, and, therefore, labels such knowledge analytic. He thinks of physical space as a matter-of-fact structure of relations about which we know a posteriori (i.e., based on experience). And, finally, he explains space in another sense as "the space of intuition" (*Anschauungsraum*): a material-essentially necessary order to which anything perceivable, hence anything falling under observed, matter-of-fact spatial relations, must conform.

[18]It derives, via Brentano, from Thomas Aquinas. Frege and Russell notoriously thought about logic quite differently, and the quasi-Hegelian logical ideas of Natorp and Cassirer are even farther removed.

To clarify the last point, recall first that the primary meaning of "intuition" (*Anschauung*), for most in this period, is ordinary sense perception (including, for some, also related acts such as imagination and memory). This results from a popular interpretation of Kant, which, while still popular today, is debatable: other interpretations (as in Natorp) yield other uses of the term. In no case, however, is *Anschauung* the name of a special faculty for knowledge about essences or about mathematics.[19] Proponents of such special faculties—including Husserl and, at this stage, Carnap—do sometimes call them *Anschauung*, but only as a conscious broadening of the term, based on some *resemblance* between the faculties in question and ordinary perception.[20] This simple terminological point has often given rise to confusion. The moral for us is just that, although Carnap in *Der Raum* does indeed believe in a faculty of essential insight, his term *Anschauungsraum* nevertheless means: the space of all possible sense perception (and imagination), not: the space revealed by a special faculty of "intuition."

On the other hand, "physical space," in *Der Raum*, is *also* a space of perception. The terminology here is not standard: in Husserl, as well as in the *Aufbau*, the contrast between intuitive or perceptual space and "physical space" is precisely that between a space of perception and the constructed (or "substructed"), mathematical, *non*-perceptual space of physics: a space which can contain geometrical shapes and/or numerical charges and field strengths, but not, for example, colored surfaces. In *Der Raum*, as we will see, both the *Anschauungsraum* and the physical space do depend essentially upon mathematical construction for their completeness, but what *gets* thus completed in each case is a space of perception: the relationship between the two, as Carnap makes clear, is that of a generic (material) essence to an instance falling under it (1922, §IV, p. 60). The *Anschauungsraum*, in other words, is the structure of non-formal, "synthetic" a priori constraints on the possible form of physical space, where the latter is the space in which objects are actually, as a matter of fact, intuited (i.e., perceived).[21] A confusion between the two meanings of the

[19]The closest I have found is in Weyl (1927), p. 93. Weyl, following Fichte, distinguishes between sensation (*Empfindung*) and intuition (*Anschauung*), and attributes our knowledge of spatial relations to the latter. But even there *Anschauung* is a non-qualitative component *of ordinary perception*. (Weyl's usage elsewhere is more standard.)

[20]See Husserl, (1976 [1913]), §3, p. 11/Hua 3.1:14,25–6: "*Auch* Wesenserschauung ist eben Anschauung"; echoed by Carnap, (1922), §II, p. 23: "Im Allgemeinem mag aber der Ausdruck Anschauung *auch* die Wesenserschauung mit umfassen" (my emphasis in both cases).

[21]See also Carnap's analogy (ibid.): the relationship between *Anschauungsraum* and physical space is like that between the rule "Three groups of four things [*Dinge*] each contain just as many things as four groups of three things" and the inference "The number [*Anzahl*] of

phrase is perhaps behind the interpretation of *Der Raum* which one sometimes hears, according to which the *Anschauungsraum* is Euclidean while physical space is not. Such a view would make perfect sense if "physical space" were understood as it is in the *Aufbau*, but, besides contradicting all of Carnap's explicit statements about the *Anschauungsraum*, it is also clearly incompatible with the idea that the *Aunschauungsraum* is to physical space as generic essence to individual.

The three-way distinction between formal possibility, *anschauungsmäßig* possibility, and (physical) actuality is not unique to Husserl. Such classifications are widespread among the philosophically minded mathematicians mentioned above: found, for example, in Killing, Hausdorff, and Frege.[22] But these authors have no full-blown epistemological system, and Carnap never mentions them (Frege included) in general epistemological contexts. There are also some points of similarity with Driesch (in particular with regard to the importance of *Sosein*, to be discussed later). But Driesch's complete view is utterly different from Carnap's and Husserl's. For Driesch, spatiality is one of the fundamental and irreducible order-constituents of consciousness (*Ursetzungen*) (1912, p. 82). From a Husserlian point of view, it impossibly combines aspects of the formal and the material eidetic (not to mention the noetic and the noematic, as well as the immanent and the transcendent). In any case, Carnap's particular version of this three-way classification is distinctively Husserlian. Carnap differs from Husserl, however, on one important point: he allows that non-Euclidean spaces are not only formally, but *anschauungsmäßig* possible.

2 Husserlian problems about general relativity, and Carnap's quasi-Husserlian solution

The disagreement is clearly motivated by difficulties concerning general relativity. But the form in which Carnap confronts these difficulties is itself revealing. Euclidean geometry had served, historically, as a paradigm of necessary truth, so nearly everyone was troubled by claims of possible or actual alternatives. But the nature of the trouble varies. In a Marburg neo-Kantian system such as Natorp's, for example, mathematical/scientific theory advances via discov-

these bins here is three, and the number of balls in each is four; there there are four bins and three balls in each; therefore there are just as many balls here as there." The balls in the second statement are individual instances of the material genus *Ding* (perceivable thing) for which an a priori law is established in the first statement.

[22] See Killing (1885), p. iii; Hausdorff (1904), p. 3; Frege, (1884), pp. 20–21.

ery of more and more determinative conditions on the possibility of objective givenness.[23] The results of this process, as logical conditions for the very possibility of evidence, ought not to be subject to empirical challenge (Natorp, 1910, p. v). The homogeneity, isotropy, and flatness of space were widely supposed to be among such conditions (pp. 226–8, 307–9). If general relativity is correct then, at a minimum, there is some error in the demonstration that just these conditions are necessary.

Husserl's problems may, in comparison, seem rather minor. It is surprising that theorems of Euclidean geometry, supposedly a matter of material-eidetic evidence, turn out to be false.[24] But Husserl, unlike Natorp, never *argues* that they are (necessarily) true. Phenomenology explains our supposed knowledge of them by referring to acts of *Wesenserschauung*—acts which, in a case like this (insight into the essence of an external object), are eminently fallible (Husserl, 1976 [1913], §149, pp. 310–11/Hua 3.1:345,18–346,2; see also §60, pp. 113–15/128–30).

A deeper problem is raised, however, by the connection between possibility and imaginability. It is supposed to be phenomenologically evident that certain transitions between *Wesenserschauung* and ordinary intuition are always possible. A rational transition from intuition (perception) of a matter of fact to intuition of an essential truth which it evidently instantiates must always be possible: we can turn directly towards the necessity of whatever is evidently necessary in some perceived state of affairs (§3, p. 12/Hua 3.1:15,29–33). But a transition in the other direction, in which an instance is produced for a rationally posited necessary law, is also always supposed to be possible—except that the instance in question may be only imaginary (p. 12/15,26–9; §4, p. 13/16,13–20). This underwrites Husserl's so-called method of free variation: the limits of what is material-eidetically possible are also the limits of what can be, if not perceived, then at least imagined. Thus if, as was widely (though not universally) thought, non-Euclidean geometries are unimaginable, they would be ruled out as descriptions of actual spatial relations.

Carnap could have avoided the first problem simply by conceding that eidetic insight does not fully settle geometry: prescribes, say, only that space be a Riemannian manifold. The material essence of space would be the deformalization of a different, slightly more general formal category. But, although the thesis of

[23]For more discussion of this, see Stone (2005) and passages cited there, especially Natorp (1910), pp. 354, 358–60, 366.

[24]Husserl's own attempts to deal with general relativity begin with this surprise: see e.g. Husserl (2002), Hua 35:297,5–7.

Der Raum is sometimes summarized this way, it is actually more complicated. This is because Carnap has the second, deeper worry in mind. Certainly he subscribes to Husserl's equation of imaginability and material-eidetic possibility. Material-eidetic insight, unlike formal-eidetic, does have a certain basis in experience: it possesses "intuitive significance" (Carnap, 1922, §I, p. 7). But this is not because its deliverances are empirical rules gathered from repeated experiences; rather, their hold over all *possible* experience can be established by free variation, based on only a single imagined example (§II, p. 22). So Carnap faces Husserl's problem. Can we, for example, imagine a two-sided polygon? And if not, why doesn't this limit on imaginative variation provide the grounds for material-eidetic insight that only Euclidean geometry is physically possible?

Carnap answers, based on a substantial understanding both of differential geometry and of Husserl, that eidetic insight reveals only the local character of the *Anschauungsraum*: its behavior in (arbitrarily) small regions. The mathematical motivation for this is that a Riemannian manifold is "infinitesimally flat": its geometry is arbitrarily close to Euclidean in a sufficiently small region around any point. If eidetic insight prescribes only infinitesimally Euclidean behavior, therefore, it will not rule out any geometry envisioned in general relativity.[25] Meanwhile, however, there are reasons to think that Husserl ought to have adopted this view in the first place.

Begin with the fact that we cannot imagine arbitrarily large regions of space. Husserl agrees with this completely. His first book (1970 [1891]) is all about the contrast between the small finite realm (*Gebiet*) of intuition and the infinite structures of mathematics. The prime example there is the series of cardinalities obtained by successively adding members to a collectivum. But Husserl also mentions, as example of something beyond the reach of our intuition, "the light-years of the astronomers" (Hua 12:192,3–4), and, as example of an infinite set which can be symbolically, but not intuitively, represented, the set of points on a line (219,12–2). These two may hint at different respects in which space escapes our capacities: infinite extension and infinite divisibility. In any case, by the time of the 1907 lectures published as *Ding und Raum*, Husserl explicitly gives both infinitudes (both familiar from Kant's Antinomies, and before that from Hume) as examples of the limits of intuition. Despite the double infinitude, he explains, "every possible presentation [*Darstellung*] must make do with limited

[25] Except that general relativity is not really a theory of space as a Riemannian three-manifold, but of space-time as a quasi-Riemannian four-manifold. Carnap is aware of this complication, but ignores it for the sake of simplicity (see §III, pp. 40–41, 46, 57, 59).

means of presentation" (1973, Hua 16:122,13–14).[26] But then Carnap is right to say, based on the connection between imaginability and essential possibility, that material eidetic insight can deliver directly only the *local* properties of space.

The question, then, is how to extend eidetic knowledge about the finite realm of intuition into a priori global knowledge about the surrounding infinite structure. Husserl is no stranger to this question. The whole plot of the *Philosophie der Arithmetik* turns around the way a limited *Gebiet* of intuitive, "proper" (*eigentlich*) representation—in this case, the representation, with respect to cardinality, of small collectiva—can be symbolically extended (*erweitert*) to infinity. Husserl answers that, although we have no intuitive access to large cardinalities, we *do* have direct access to the clearly defined operation which, given any arbitrary (finite) collectivum, yields a new one whose cardinality is one greater. The "complete extension [*Erweiterung*] of which the concept of [finite] set or multitude is capable by symbolic means" is possible, in other words, because "the process of adding-on [*Hinzufügung*] of a unity to an arbitrary given number is an operation whose concept guarantees, a priori, that it leads to a new, determinate number" (Husserl, 1970 [1891], Hua 12:218,26–8, 220,11–14). The operation of adding intermediate points between two points of a line is given there as a further example of the same principle, and in (1973) Husserl says that the spatially large or small can only be presented "through the serial order form of the operations" by which the limited means of intuition "come into action again and again" (Hua 16:122,14–18)—a process which Husserl there, too, calls *Erweiterung* (e.g., 205,15; 209,16).

Here, again, Carnap follows Husserl. The *Erweiterung* of the *Anschauunsrgaum* beyond the limited *Gebiet* of our perceptual or imaginative field takes place, he explains, through a step-by-step extrapolation of our limited imaginative space into an unlimited global structure, a *Gesamtgefüge*. We are justified in doing so, he says, because "if . . . the species of a formation permits a second one of the same species to be added on to it in a determinate manner, we can demand [*fordern*] that this adding-on [*Anfügen*] should be further possible

[26] I am not aware of any evidence that Carnap knew, by 1921, of the specific contents of these lectures. At a minimum, they show how accurate was Carnap's extrapolation from Husserl's published works. Husserl even mentions that the considerations under discussion would apply in a space of any dimensionality and curvature (122,1–4). So he is already close to Carnap's solution. Husserl does not allow for variable curvature, however, and also adds that even constant-curvature non-Euclidean spaces are inconsistent with the essence of "our" spatial perception.

without end" (Carnap, 1922, §II, p. 23).[27] So Carnap and Husserl agree, not only on the initially local nature of geometric knowledge, but also on the means for extending it. Why, then, does Husserl not agree with Carnap on the eidetic possibility of a non-Euclidean physical space?

The answer is that infinitesimal is not the same as local. Although every Riemannian manifold is everywhere infinitesimally Euclidean, only Euclidean space itself, and trivial variations thereon, are Euclidean in a small *finite* region around every point. Husserl is correct to argue, then, that local determines global structure. Since, as he puts it, the internal ordering (*Anordnung*) of the perceptual field prescribes a fixed order (*Ordnung*) to its contents, and since continuous shifting of the field involves a continuous positing of unity in the shifting images, "there arises the consciousness of a thing-manifold of fixed order, and finally of the world" (Husserl, 1973, Hua 16:217,30–35). The relation even of distant perceptible objects is fixed by the order of the intervening fields, between which in principle one might shift one's perception on the way from one to the other (218,9–12).

Carnap, conversely, must argue that eidetic insight reveals, not finitely local, but only infinitesimal structure. Unfortunately his presentation becomes vague at this crucial juncture, so that, although an argument can easily be reconstructed, it is difficult to be sure what he had in mind.[28] One possibility would be to claim that the realm of intuition (perception or imagination) is itself infinitesimal.[29] But that sounds implausible, if indeed it makes any sense at all. Carnap probably means, instead, to go in the opposite direction. For, as Husserl would notice somewhat later, the fixed extension of local geometry into the arbitrarily large is blocked thanks to the other limit of perception, in the the arbitrarily small.[30] Perception gives only limited detail: it is always only of relative straightness, equality, perpendicularity, etc.[31] We can imagine indefi-

[27]For the terms *Erweiterung* and *Gebiet*, see Literatur-Hinweise, p. 81 (note to p. 26). This terminology is also found in other authors, however (including Killing and Pasch), so it is not unequivocal evidence of Husserl's influence. On *fordern* and *Gefüge*, see further below.

[28]For the view that Carnap was simply confused, see Sarkar (2003), p. 188.

[29]This appears to be Friedman's interpretation: see (1995), p. 48.

[30]See Husserl (1954), Abhandlung titled "Realitätswissenschaft und Idealisierung: Die Mathematisierung der Natur" (dated by Biemel to 1926–8), 290,21–6, and see also Beilage III (the 1936 text published by Fink as "Der Ursprung der Geometrie"), 384,10–45.

[31]Cf. Hume (1978) 1.2.4, pp. 49–50. Husserl gave a seminar on Part I of the *Treatise*, in conjunction with his lectures on the history of modern philosophy, in the summer of 1921 (Schuhmann, 1977, p. 246), at a time when Carnap was living near Freiburg and completing work on *Der Raum*. (Unfortunately the Husserl Archive has no manuscript of these 1921

nite improvements on this—focusing in, so to speak, more and more closely on a smaller and smaller region—but an *infinitely* detailed view is unimaginable. Even if imagined spatial relations are always perfectly Euclidean, then, that just means that they agree with Euclidean axioms up to the level of detail that is actually imagined, and the corresponding eidetic knowledge is just that space must be arbitrarily close to Euclidean in sufficiently small regions.[32] This argument, though not explicitly in Carnap, does fit well with the way he talks: he constantly mentions small regions of space, without saying exactly how small, and never mentions infinitesimal "regions" at all. It is also suggested by his technical sources, the most important in this respect being Killing, Pasch, and Klein.[33]

Mathematically, Carnap is closest to Killing, who begins with a subset of what he takes to be Euclid's assumptions, leaving out the non-local parallel axiom and the assumption that the straight line is infinite. On this basis, Killing attempts to show, first, that all of Euclid's theorems hold in a "limited *Gebiet*." This turns out to mean that they hold arbitrarily well in every sufficiently small neighborhood—or, as Killing more typically says, that they hold perfectly in infinitely small neighborhoods.[34] Four global possibilities—Euclidean, hyperbolic, and single or double elliptic geometry—are then shown to follow from different ways of extending local (infinitesimal) segments into geodesics. Carnap updates this approach mostly by substituting Hilbert's axioms for Euclid's. He also claims to include the general case of variable curvature, though without enough mathematical detail to determine how or whether that is possible.[35]

lectures: see Goossens's introduction to Husserl (2002), p. xv n. 1.)

[32]Assuming that imagined objects have no intrinsic scale. See Husserl (1973), Hua 16:121,19–33, and cf. Hume (1978), 1.2.1, p. 28.

[33]For the first two, see Killing (1885), pp. 1–17, and Pasch (1912), pp. 4–20, both cited by Carnap (1922), Literatur-Hinweise, p. 81 (note to p. 26). Klein is not mentioned in that important note, but Carnap cites him twice on p. 80 (notes to p. 23), in connection with "the inexactness of intuition" and "the limitedness of the *Anschauungsgebiet*."

[34]Killing alternates between these two types of formulation. See e.g. (1885, p. 3), where the main theorem, "In every triangle whose sides are, all taken together, infinitely small, the sum of the angles equals two right angles," is followed by a long paraphrase in terms of allowing a given finite triangle to shrink, etc.

[35]Killing's own original intention had been to take on the general case, but he decided against it as too large a project (1885, p. vi). The technical result Carnap probably would have needed to back up his claims, Weyl (1921), was published too late for Carnap to have seen it (although he does cite Weyl's earlier work, and recommends Weyl as the primary source to consult on general relativity). See the discussion in Torretti (1996), pp. 191–4. Carnap also allows another generalization, to n dimensions (see 1922, §II, p. 30), though he does

Epistemologically, however, Carnap is closer to Pasch, who explicitly claims to derive his axioms from our finite capabilities of perception and imagination (1912, pp. 14, 17) and to Klein, who follows Pasch in this respect.[36] Pasch and Klein, unlike Killing, never speak of infinitely small regions in which the Euclidean axioms hold perfectly. And whereas Killing, in explaining why we cannot determine the precise geometry of physical space, appeals to the limits of measurement devices (1885, p. 13), Pasch and Klein see a limitation intrinsic to our form of intuition. This presumably is why Carnap criticizes Killing on this issue, but not Pasch or Klein (1922, Literatur-Hinweise, p. 83 [note to p. 54]). The absence of a determinate metric and affine structure could not be supplied, as Killing implies it could, even by infinitely improved physical measurement (as if an infinitely straight and regularly marked ruler could be used to establish the very definitions of "straight" and "regularly marked"). But if *imagination* were infinitely precise then, on the Husserlian view Carnap takes, the affine and metric structures would be determined in advance of all physical measurement. This is because imagined pictures, on this view, include an irreducible phenomenological character of, for example, straightness: one would not need to make any measurement to determine whether one is *imagining* a straight line. The key point, then, is that the straightness of such a line, like every other feature of the imagined situation, is never given with infinite precision.

Whatever the details of the argument, Carnap's conclusion is that eidetic insight founded on our actual imaginative capabilities is *not* sufficient to fix global geometry. It is important to understand the depth of the problem. If eidetic insight fails to prescribe a determinate global geometry, then it fails to

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think we know a priori that $n \geq 3$ (§V, pp. 66–7). He discusses this issue only briefly, since he knows of no empirical evidence best accommodated via more than three spatial dimensions (see §III, p. 38). But he was open to the possibility, as can be seen from his reference to Kaluza in (1924), p. 117 n. 1. It is due to this second generalization that Carnap ends up concluding that the synthetic a priori structure of space is that of "topological intuitive space with indefinitely many dimensions (R'_{nt})" (1922, §V, p. 67): in other words, the topological structure of a general Riemannian n-manifold, considered in abstraction from its affine or metric properties.

[36] The first source which Carnap lists on "inexactness of intuition" (Carnap, 1922, Literatur-Hinweise, p. 80) is Klein (1890), p. 571 n., and on that very page Klein himself immediately refers the reader to Pasch. It is unclear to me why Carnap cites Klein's (1914) only on the finiteness of the realm of imagination (hence the impossibility of determining a priori whether a line can be infinitely extended) and not on its inexactness (Klein in fact discusses both there), or why he cites Pasch directly on the first point but not on the second.

fix any relation at all between distant objects, and hence leaves us with no global intuitive space whatsoever. We get no a priori reason to think, for example, that a thing outside the limits of our perception must be *somewhere* (must lie in some determinate spatial relation to the perceived objects). Yet mathematical physics makes constant use of the assumption that some determinate geometry applies to arbitrarily large regions, or even, in cosmology, to space as a whole.[37] If eidetic insight does not justify the assumption that physical space, though known to us only through imprecise perception, nevertheless instantiates some determinate geometrical structure—then what does? Husserl himself ultimately concluded that this assumption, along with others which go into Galileo's "mathematization of nature," is indeed unjustifiable.[38] But Carnap instead, fatefully, appeals to the notion of a *Forderung*—a "demand" or "postulate."[39] Thus he transforms the question from: to what extent does eidetic insight leave geometry undetermined? to: what a priori conditions must we demand of possible experience, *beyond* those derivable from eidetic insight, in order to guarantee that it conform to *some* global spatial order?

3 Carnap's use of Driesch, Dingler, and Poincaré

Even this has Husserlian precedent: the term *Forderung* is found in relevant contexts in (1970 [1891]).[40] But Carnap's main source in this respect is Driesch, whose *Ordnungslehre* is built around the concepts of *Forderung* and *Gefüge*. The full system is complicated and not entirely coherent, but the relevant points can be summarized as follows.

All philosophy, according to Driesch, begins with reflection upon the stream of experience. Nothing could be said about this, however, if there it were not for certain "order constituents" in the stream thus revealed (1912, p. 6). An order constituent, as expression of "thought," is a sign of "conclusiveness" (*Endgültigkeit*) (p. 5). It establishes that what has been experienced has been experienced *as* such-and-such, an "as" which *ought* to hold for all further experience, as well. The second part of philosophy, *Ordnungslehre*, thus arises as "the theory of con-

[37] Actually, GR as ordinarily understood assigns to space(-time), not a geometry, but an equivalence class of geometries. Again, Carnap ignores this complication.

[38] See (1954), §9h, Hua 6:48–54.

[39] *Forderung*, in a mathematical contexts, means "postulate." As will become clear, however, Carnap is interested also in the term's non-technical force, as "demand."

[40] See, e.g., Hua 12:199,9–11: "doch steht nichts im Wege, dem Begriffe des Processes die Forderung beizufügen, daß er alle erdenklichen Glieder in sich aufnehme."

clusiveness on the grounds of a methodological solipsism" (p. 8).

"Thought," then, is what first establishes an ought; its "primordial achievement" (*Urleistung*) is a demand, a *Forderung*, of order (p. 7). Since every order constituent presents itself as the fulfillment of this demand (p. 18), thought fulfills its own demands in the act of making them. It cannot be constrained by any order, because it first produces and recognizes order: all order is its "free achievement" (p. 34). Nevertheless, according to Driesch, there is a kind of *Forderung* which reflects back on the ordering ego itself, an "ought to posit positings which ought to hold" (p. 6). This special type of *Forderung* is "the self-demand of *parsimony of positings*" (p. 110). The general idea behind it is that, where various order constituents can be understood as connected by a rule, the number of independent ones can and should be reduced by considering them as quasi-logically connected, "as if co-posited" with one another (p. 146). Driesch first discusses this at length in his section on space—the very section which Carnap cites in *Der Raum*. But the full importance of parsimony comes out only in the realm of natural actuality, where it results in the positing of objects "as if" independent of the ego (pp. 161–2). This can be accomplished because, in a certain selected piece, a certain *Ausschnitt*, of experience (p. 132), later experiences can be taken as as if co-posited with earlier ones: namely, insofar as both can be interpreted as the appearance of physically actual objects at different points in their process of becoming. (The remaining experiences, outside this *Ausschnitt*, are then dismissed as dreams, hallucinations, etc.)

Now a "system," or *Gefüge*, for Driesch, is an ordering-type in which each element implies all the others, so that the generic character of the *Gefüge* already predetermines all the different ways it can be instantiated (pp. 93, 121).[41] Nature, therefore, from the point of view of *Ordnungslehre*, is just a selection of experiences in which "a particular *Gefüge* of ordering thought-demands is found to be *fulfilled*: a *Gefüge* of demands with respect to *becoming*" (Driesch, 1912, p. 132).

This selection of experience is in turn possible, however, only thanks to more fundamental types of *Gefüge*. Suppose, to take Driesch's example, that I experience three houses. This counts as a *correct* experience only if it does not contradict the unified positing of nature. A contradiction *can* arise here

[41] Driesch indicates several times in the text (pp. 87, 93, 138) and also in the index (p. 344) that *Gefüge* is a replacement for the term *System* (in line with his general policy of avoiding Greek and Latin terms: see p. 10). Cf. Mormann (2003), p. 47. Mormann is puzzled by Carnap's use of the term *Ordnungsgefüge*, noting that it is "not a terminus technicus in mathematics."

only thanks to the status of the natural world as *Gefüge*: my other experiences, taken together, may quasi-logically imply, say, exactly two houses—two houses (and no more) may be "as if co-posited" with them. But three houses don't per se contradict anything: there can be three on this street and two on the next. To say that an experience is correct is to say that "*here* and *now*, that is, in these points of the space of nature ... and the time of nature ..., *this such* exists with natural actuality" (p. 160). The possibility of contradiction also depends, in other words, on the unified order of space and time. And the unified structure of space is itself a *Gefüge*. For example: there is no contradiction in the idea of a two-sided polygon; only, in Euclidean space, the third side is as if co-posited with the other two. This spatial *Gefüge* must therefore be the result of an earlier and more fundamental demand for parsimony.

It is in these terms that Carnap justifies his own global *Anschauungsraum*, as the *Gefüge* answering to the a priori *Forderung* of global spatial structure. He is following Driesch, moreover, as well as Husserl, in taking this as a demand that a serial "adding-on" should be completable in a determinate way.[42] In Driesch's terms: while the primitive phenomenological character or *Sosein* of spatiality directly yields a list of local axioms, a *Forderung* of thought is needed to back up the judgment that "segments, planes, and parts of space can be increased by a *so much* through adding-on others of the same kind" (Driesch, 1912, p. 109). Carnap simply gives this a more precise technical content, based on Killing. Finally, Carnap and Driesch even agree that there is more than one way we could impose the demand: "a genuine choice between several conclusivenesses which present themselves as possible to adhere to" (pp. 109–10). They disagree only about how, and at what stage, we are to choose.

For Driesch, further demands of *geometrical* parsimony require space to be Euclidean (pp. 111–16). Carnap, however, while agreeing that the "spatial system" (*Raumgefüge*) is ultimately determined by further demands, maintains they must not be added to our positing of the *Anschauungsraum*, i.e. to the a priori, purely geometrical structure of space, as posited in advance of all experience. A priori *Forderungen*, rather, in his view, must not go beyond the bare requirement of *some* (Riemmanian) global structure. This is not due to lack of theoretical justification: if eidetic insight doesn't justify the demand for any global structure at all, then we would be no *less* justified in adding a further

[42]See the important note already cited several times above: Carnap (1922), Literatur-Hinweise, p. 81 (note to p. 26), where Carnap lists sources to consult on "the *Erweiterung* of the spatial *Gebiet*." Besides Killing, Pasch, and Driesch, the list includes only Benno Kerry's vaguely relevant (1890).

Forderung of Euclideanness (Carnap, 1922, §II, p. 28). Rather, the problem is practical. The spatial *Gefüge* is itself only posited, as Driesch would agree, for the sake of what comes next, the construction of a "contradiction-free *Gefüge*" of physical actuality (§IV, p. 61). What the overall demand of parsimony requires here, then, is simplicity, not in geometrical determinations per se, but only in "the structure which follows on the ground of those determinations" (§III, p. 55). If we were free to impose simplicity at the purely geometrical stage, Carnap agrees, we would demand Euclidean order. But this demand would interfere with the ultimate goal of ordering natural actuality, and is therefore *practically* ruled out. What holds of Carnap's flat-earth metric (§III, p. 52) holds also, in a subtler way, of an a priori demand for Euclidean space: we cannot seriously (*ernstlich*) choose it.

This brings us to Carnap's use of the "conventionalists" Dingler and Poincaré. Of the two, one might well expect Dingler to be the more important. Although Carnap later (1963b, p. 15) lists both Dingler and Poincaré as important conventionalist influences, albeit with respect to a slightly later stage of his thought, Dingler alone turns up as the primary source to consult in Carnap's general note to §III of (1922) (Literatur-Hinweise, p. 82). Moreover, as Wolters (1985) demonstrates, Carnap and Dingler had extensive personal contacts during the time when Carnap was at work on *Der Raum*. In fact, however, Dingler's influence is rather difficult to trace. Dingler's two central ideas are (1) that science approximates objective reality by fitting it into a series of progressively more exacting subjectively imposed frameworks, in a procedure analogous to Fourier analysis of a function (his "principle of exhaustion") and (2) that the contact between our "manual" dealings with reality and our theoretical attempts to describe it occurs primarily at the sites where precision instruments of measurement are produced. For Dingler, (1) and (2) put together serve to rule out the possible correctness of GR. Dingler admits that one might start the series with a different "null point" — in particular, with Einstein's definition of simultaneity, rather than with a definition of length in terms of (Euclidean) rigid bodies;[43] this is analogous to the fact that one might replace the sine and cosine functions of Fourier analysis with some other basis (see Dingler, 1911, pp. 56–7). But to make such a choice in order to solve particular problems in physics would, according to Dingler, undermine the whole process of "exhaustion" (something like simplifying the series expansion of a

[43]Dingler (1921), p. 164; cited out of context by Carnap (1922), Literatur-Hinweise, p. 84 (note 2b to p. 54).

function f by choosing a new basis in which f itself is one of the basis functions). And, therefore, according to Dingler, those who actually carry out the analysis of reality—namely, the artisan-engineer-scientists who produce precision instruments—will never accede to such a choice (1921, pp. 32–3). Of all this there is no trace, either positive or negative, in Carnap (and therefore no sign of understanding that Dingler's thought cannot be reconciled with relativity via the type of modest corrections which Carnap applies to Husserl and Driesch). The most characteristically Dingler-like point in Carnap (1922) is instead Carnap's argument, at the beginning of §III, to the effect that only a freely chosen stipulation can cut off the infinite regress in demands for justification (p. 33; cf. Dingler, 1921, pp. 99–100, 127). But this could hardly represent Carnap's complete motivation for introducing stipulative demands or conventions, since, unlike Dingler, he does not accept the argument in full generality: on the contrary, he is willing to let the regress of justification be cut off by appeal to intuition, whether ordinary empirical intuition or (at this stage) *Wesenserschauung*.[44] This helps to explain in what sense Carnap in (1963b) refers to Dingler as a "radical" conventionalist, and why he there limits Dingler's important influence on himself to some papers written between *Der Raum* and the *Aufbau*. Here in (1922), it seems, Carnap refers us to him, in connection with §III only, mainly just as one among the many who argue that empirical facts alone do not force a choice of geometry. As for why he singles Dingler out for special notice, we may speculate that this was a courtesy in light of the friendly personal relationship which Wolters describes.

The influence of Poincaré goes somewhat deeper. Carnap and Poincaré are working, true, towards different goals, based on different assumptions. Poincaré's idea of the synthetic a priori, for example, has little to do with Husserl's. Moreover, Carnap is far more radical about space, claiming that not only our particular geometry, but our notion of global spatiality as such, rests on a choice made in response to the demand of parsimony—on a convention, if you like. All Carnap wants from Poincaré, however, are the means to fill in a very specific place in his own project. Having argued, against Husserl, that the limits of imaginability do not establish a full global structure for space, and having argued, against Driesch, that the demand for geometrical parsimony must yield to that for physical parsimony, he must now demonstrate that the

[44]This is the core of what Popper would later call Carnap's "justificationism." The special vehemence with which Popper attacks Dingler, on the other hand, is due to the fact that he and Dingler are so close to one another on this point.

two demands can actually conflict, and explain how the overriding demand of physical parsimony can be sufficient to impose a full affine and metric structure. In §III of *Der Raum*, Carnap turns to Poincaré for help with those limited tasks.

Poincaré might seem an unlikely ally, since he, as much as Dingler, appears ultimately to conclude that no experiment will ever get us to choose a non-Euclidean geometry. But Carnap sees that conclusion as resting on an assumption about the contingent, empirical behavior of our world: given other circumstances, which Poincaré thought counterfactual, the same considerations lead to a different choice (Carnap, 1922, §III, p. 56). Whether or not this view is correctly imputed to Poincaré, moreover,[45] his examples help make the point vivid. To allude to the most famous one: hyperbolic geometry seems simpler than Euclidean geometry plus specially rigged thermal gradients and laws of refraction.[46] What Carnap wants from these examples is that the means for establishing geometrical "matters of fact" are always physical: their results will vary with changes of view about the physical laws applying to the measuring instruments. Carnap thus takes his cue from Poincaré in pointing out how changing our theory of thermal expansion could change the result of measuring with a rod, and changing our theory of refraction could change the result of establishing straightness by line of sight—i.e., how metric and affine *Tatsachen* would have to change to accommodate different physical theories.[47] Given such interchanges between physical and geometrical, an intrinsically more parsimonious geometry might be purchasable only at the cost of a less parsimonious total theory, so that Driesch is, at least in principle, incorrect. Which global geometry *ought* to be demanded will depend on empirical matters of fact.

[45]He appears to make a stronger, psychological prediction that *we*, given our upbringing, will always continue to use Euclidean geometry, even if we turn out to live in a world where some other geometry would be more convenient: see (1906), pp. 52, 73, 74–5.

[46]See (Poincaré, 1906), pp. 66–70. Not that I know of a rigorous definition of relative simplicity from which this would follow. At the time, however, Driesch and many others (e.g., Dingler) believed that the concept of simplicity was, or would soon be, sufficiently understood, and Carnap apparently agrees.

[47]See (Carnap, 1922), §III, pp. 34–6, 52. That these examples require strange idealizing assumptions (e.g. that thermal expansion is instantaneous, and need not be accompanied by other effects such as black-body radiation) only serves to strengthen the impression that Carnap is following Poincaré, rather than coming up with his own examples. In the flat-earth example, which really is Carnap's, he makes no attempt to tie the required law of length change to a familiar mechanism such as thermal expansion.

4 Implications for Carnap's later thought

Let me now turn back, briefly, to the forward-looking implications which I mentioned to begin with.

In the *Aufbau*, Carnap characterizes his position in various ways. He opposes traditional "metaphysics"; rejects the synthetic a priori; rejects "problems of essence"; considers meaningless all terms without finite empirical criteria of application; believes that all meaningful, scientific statements are "structural descriptions"; and holds, as his "main thesis" (*Hauptthese*), that "there is only one realm of objects and therefore only one science" (1974 [1928], §4, p. 4). Now, to give a structural description (of a relation) is to give its "formal properties" (§11, p. 13). That only such formal properties are intersubjectively communicable is a thesis which Husserl defends in *Ideen* II, and for the same reason given by Carnap: the qualitative contents of different streams of consciousness are incommensurable. The perceived thing as intersubjective, Husserl concludes, "has no sensuous-intuitive content whatsoever," but is rather

> only *an empty identical something* as a correlate of the identification, possible according to and grounded by experiential-logical rules, of what appears, in changing "appearances" of different content, to the subjects which stand in intersubjective connection.[48]

Recall, however, that *formal* descriptions include nothing peculiar to any region of being: the formal-eidetic concerns only the form of a region in general. To communicate the nature of (intuitive) space, for example, one would need to discuss, not only the formal properties of spatial relations, but also their material essence. This is why Carnap says, in *Der Raum*, that the "particular being-thus [*Sosein*]" of the *Anschauungsraum* cannot be "conceptually defined in a formal way"; it can only be communicated by "indicating contents of experience" (*auf Erlebnisinhalte hinweisen*) (see Carnap, 1922, §II, pp. 22, 24). The term *Sosein* is Drieschian, and the view is indeed Driesch's (see Driesch, 1912,

[48]Husserl (1952), §18g, Hua 4:88,26–9. Although this work (*Ideen* II) was published only posthumously, it is very likely that Carnap saw the text (as has been argued by Haddock (2008), p. 47–8, and suggested by Richardson (2003), p. 175). Husserl encouraged his students to read such manuscript material, and even assumed that they were familiar with it. Moreover, the period during which Carnap was writing the initial draft of the *Aufbau*, attending Husserl's seminar, and discussing philosophical issues with Landgrebe, coincides with the period when Landgrebe edited the text of *Ideen* II. Hence Carnap had easy access to it and would have known, from Landgrebe, about its contents: "constitution"; nature and *Geist*; intersubjectivity.

p. 109). But Husserl is also committed to it—again, because the regional, material-eidetic characterization of a relation necessarily goes beyond its formal characterization—and, if he and Carnap are right that the intersubjectively communicable is purely formal, they will therefore both end up claiming to talk about the incommunicable. Or, to put the same conclusions together in the opposite direction: if all meaningful utterances are intersubjectively verifiable, they must all be structural (purely formal); hence cannot describe (material) essences (solutions to "problems of essence"); hence cannot describe objects as falling into more than one region. These difficult topics require more detailed textual and philosophical analysis, but it seems, at the very least, worth entertaining the thought that the synthetic a priori rejected in the *Aufbau* is the very Husserlian synthetic a priori accepted in *Der Raum*, and that the "metaphysics" rejected there is, paradigmatically, Husserlian phenomenology.

On the other hand, *Der Raum* also introduces strategies which carry forward into the *Aufbau* and beyond. The most important is what Richard Jeffrey has called Carnap's "voluntarism" (1991, p. 28). The term refers to the primacy of the will, in this case in accounting for the possibility of knowledge. As we have seen, Carnap, following Driesch (as well as, perhaps, Dingler), departs from Husserl in *Der Raum* precisely by letting the will partly replace Husserlian essential insight. Any theorems about the *Anschauungsraum* which are "dependent on the *Forderungen*, on the grounds of which the full *Gefüge* of the *Anschauungsraum* results," are now seen as derived from "determinations which are not matters of a priori knowledge, because they are not items of knowledge at all, but rather of convention" (Carnap, 1922, §V, p. 64). This replacement of a priori knowledge by an *Erkenntnisquelle* which is not itself *Erkenntnis* is completed in the *Aufbau*, where Carnap maintains that "there are in knowledge no components other than these two: the conventional and the empirical; hence no a priori-synthetic" (Carnap, 1974 [1928], §179, p. 253). Practical commitment—freely chosen convention—entirely takes over the role of the analytic and synthetic a priori in creating a framework within which logical deduction and empirical observation become possible. That picture, along with the strict theoretical/practical distinction on which it is based, remains central to Carnap's thought from then on, most famously (but by no means only) as the distinction, in "Empiricism, Semantics and Ontology," between internal questions, which are to be answered via theoretical judgments, and external ones, which are really practical issues in disguise, and can be "answered" only via a decision. Carnap consistently regards "metaphysics" as a result of mistaking these fundamental decisions for theoretical theses.

It could be argued that these practical commitments retain the same function also in detail. The basic problem seems always the one Carnap inherits from Husserl and Driesch, namely that of expanding the range of concepts beyond the small *Gebiet* of what is properly intuited. But changes in Carnap's other positions—beginning with the rejection of *Wesenserschauung* after *Der Raum*—make this continuity difficult to trace, and I will not argue for it here. Let me instead focus briefly on another theme: that of Carnap's so-called pragmatism.

The term "voluntarism," historically associated with theological views about the absolute will of God, tends to suggest that our freedom to make practical commitments (conventions, demands) is the freedom of indifference. That is not Driesch's view, or Poincaré's, or Dingler's, and it is not Carnap's view in *Der Raum*: the choice of conventions, he writes there, is "free and independent of experience" only in the sense that "the experiential facts cannot constrain us" to adopt one or another; "the choice, however, is not arbitrary [*willkürlich*], but rather guided by principles [*Grundsätze*]"—for example, the "teleological and methodological principle [*Prinzip*]" of simplicity (Carnap, 1922, §III, pp. 36, 56). The *Aufbau* continues in this same vein: the constitutional process is guided by general rules "which can be designated as a priori rules insofar as the constitution and knowledge of objects logically rests upon them," but which "are not to be designated as 'a priori knowledge' because they represent, not knowledge, but rather *convention*" (Carnap, 1974 [1928], §103, pp. 143–4). Moreover, he suggests that all these rules might be derivable "from a supreme principle [*Prinzip*]," which would in turn then need to be derived "from what is achieved by knowing [*Erkennen*] for the more inclusive teleological nexus [*Zwecksummenhang*] of life" (§105, pp. 145–6). Moreover, I (like Jeffrey) do not believe that Carnap, even later, ever adopted an arbitrary voluntarism. The oft-cited dictum of the *Logical Syntax*, that "in logic, there are no morals," is sometimes read as such a move. But it is no such thing, as is shown even by the immediate continuation:

> Everyone may construct his logic, i.e. form of language, as he wishes. Only, *if he wishes to discuss it with us, he must clearly report* how he intends to do so, and give syntactical determinations instead of philosophical arguments [*Erörterungen*]. (Carnap, 1934, §17, p. 45; my emphasis)

Anyone can choose any language, but not every language is suitable for any purpose—not, for example, for the purpose discussing forms of language. The aim of the book, in fact, is the same as the *Aufbau*'s: "to provide a structure of concepts [*Begriffsgebäude*], a language" (Foreword, p. iii) which eliminates unclarity and inexactness and thus exposes pseudo-questions (p. v).[49]

To label as *pragmatist* this form of principle-guided voluntarism is to make a claim about the principles by which the will can be determined. Pragmatism is, strictly, the view that all such principles are pragmatic, rather than (purely) practical: that there are only hypothetical, not categorical imperatives. I believe that the now-standard reading of Carnap as a pragmatist does involve such a claim, although not all who uphold it have given the matter much thought.[50] If this is correct, then all practical principles are relative to interest, and can only be determined once our interests are known—presumably, a posteriori. For a voluntarist, who sees practical commitment as prerequisite to all knowledge, circularity threatens, and indeed Carnap himself, in the *Aufbau*, describes fundamental practical principles as "a priori." Still, the circle is not obviously vicious; I will not try to close the case against Carnapian pragmatism here.[51] I do want to point out that, at a minimum, the interests involved are, already in *Der Raum*, extremely abstract and universal: they include "our" interest in the unity of the world and the possibility of objective knowledge. Later, Carnap adds, or substitutes, our interest in the possibility of meaningful communication and of mutually responsible community. It is as if an engineer were asked to design, not a bridge, but the possibility of transport (indeed, the possibility of parallel transport is one of things which the *Forderungen* of *Der Raum* are supposed to establish). Are the "we" who have such interests to be distinguished from rational beings as such? And is the *Zweckzusammenhang* to which they contribute to be distinguished from the Kantian kingdom of ends?

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[49]Cf. Carnap, 1974, Foreword to the Second Edition, p. xvii.

[50]For one who has, quite carefully, see Richardson (2007), especially p. 298 n. 3 and p. 302 n. 7.

[51]Jeffrey (loc. cit.) speaks of conventions as "chosen means to our chosen ends," which sounds pragmatist, but he also says that, for Carnap, deliberate language choice is "a duty we owe ourselves as a corollary of freedom," which sounds rather Kantian.

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